University of East Anglia

Seminar on Mathematics Education
Institute of Mathematics
Pedagogical University of Kraków

@mathtask

The MathTASK programme:

Using specific classroom situations to develop mathematics teachers' capacity in identifying, interpreting and acting upon students' needs

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Research in Mathematics Education Group - UEA
https://www.uea.ac.uk/groups-and-centres/research-in-mathematics-education-group
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MathTASK is a programme of the Research in Mathematics Education group (since 2003) at the University of East Anglia

RME group
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Plan

Introduction to MathTASK (with an example)
Theoretical principles – Theoretical constructs
Work on a classroom situation – Simplification mathtask
Discussion

What is **MathTASK?**

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Mathematics teachers want their students to understand, appreciate and enjoy mathematics.

Often though, what they face in the classroom is nowhere near these high aspirations.

Students’ responses may not make sense, addressing individual needs is difficult, the class does not cooperate, technology is confusing and the resources not exactly what is needed.

What is MathTASK?

- In MathTASK, we set out from research evidence, which shows that:
  - engaging teachers with **fictional challenging situations** before they actually face them in the classroom is an effective start and
  - a discussion on specific classroom situations is more productive than a theoretical and abstract conversation
- To this aim, we design **situation-specific tasks -** which we call **mathtaks -** that emulate challenging classroom situations and we engage teachers with these tasks individually and in groups, in writing and in discussions and in a productively reflective setting of school or university-based workshops.
- **Mathtasks** always start with a **mathematical problem** and a **critical incident in the classroom** that occurs as students and teacher are working towards solving this problem.

Let’s see a ‘**mathtask**’!

The problem

“Can you make the two columns of numbers below add up to the same total by swapping just two numbers between the columns? Explain why or why not.”

1	7
3	2
8	4
5	9

The *Reasoning* mathtask

In your Year 7 lesson, you asked the students to solve the following problem:

"Can you make the two columns of numbers below add up to the same total by swapping just two numbers between the columns? Explain why or why not."

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The following conversation between students takes place:

Khalil: I will start swapping numbers and see what happens! If I try all the pairs, I will figure out what the right response is. If we swap 1 and 7, we get new totals of 23 and 16. This does not work, let me try another pair …

Mel: Hold on, if I add up the columns, I get totals of, umm…17 and 22. So we need to replace a number in the left column with a bigger number from the right column…what about swapping 5 and 7? It gives 19 and 20, closer but not right …

Wendy: Maybe it can't be done. The difference of the two columns is 5 and I have to split the difference in two equal parts. Is this possible?

Callum: We will spend all this time swapping numbers for a problem that has no solution. This is rubbish, not maths.

You have just heard this exchange between students.

Questions:

a. Solve this mathematical problem bearing in mind that this is a Y7 lesson.

b. How would you respond to Khalil, Mel, Wendy and Callum and to the whole class?

c. How would you use this problem as an opportunity to develop reasoning skills in your class (in Year 7 or other)? Can you think of other problems of this type?

A mathematical problem

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A dialogue between students who try to solve the problem

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Teachers are invited to reflect on the situation

Questions:

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MathTASK: Design Principles

- The **mathematical content** of the task concerns a topic or an issue that is known for its subtlety or for causing difficulty to students (from literature and/or previous experience) - ***Mathematically Significant Pedagogical Opportunities to build on Student Thinking*** (*MOSTs,* Leatham, Peterson, Stockero &Van Zoest, 2015)
- The **student response** reflects this **subtlety (or lack of)** or **difficulty** and provides an **opportunity** for the teacher to reflect on and demonstrate the ways in which s/he would help the student achieve subtlety or overcome difficulty (*MOSTs: pedagogical opportunity*)
- The **teacher's pedagogical approach** concerns **mathematical, pedagogical and epistemological issues** that are known for their subtlety or for being challenging to teachers - *Practical Rationality of Teaching (PRT,* Herbst and Chazan 2003); *Spectrum of Warrants* (*SW, Nardi et al. 2012*).
- **Mathematical content and student/teacher responses** provide a context in **which teachers' knowledge, beliefs and intended practices** (mathematical, pedagogical and epistemological) are allowed to surface - *Mathematical Knowledge for Teaching* (*MKT,* Hill and Ball 2004); *Horizon Knowledge* (*HK*, Ball, Thames & Phelps, 2008); *Knowledge Quartet* (KQ, Turner & Rowland 2011).

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Year 7 has been chosen to allow a discussion on student transition from primary to secondary.
Alternatively, the "Year" can stay open and participating teachers can choose the educational level of their choice. The problem can be approached differently in different educational levels.

The problem has been chosen because:

- It can by approached by both primary and secondary teachers/students
- It has the potential for expansion from elementary arithmetic to generalisation of number facts (e.g., odd and even numbers)
- Finding the right pair of numbers, if it exists, triggers exploration routines, which vary across educational levels and within educational levels (see student responses).
- The “explain why or why not” part triggers substantiation routines (reasoning), which vary across educational levels and within educational levels (see student responses).
- The problem does not have a solution, which challenges the view that every mathematical problem should have a solution.
- The value of the problem relates to the explorations and substantiations rather to performing a routine that will give a result.
- The problem has the potential of further generalisations (or generation of other problems) by teachers and students, for example:
  - Is the problem ‘solvable’, if the total of the columns is an even number?
  - Can we change the numbers to make the problem solvable?
  - Can we change the numbers in order to have more than one solutions?
- The variation of alternative problems above can serve purposes of differentiation in the mathematics classroom.

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Mel:	Hold on, if I add up the columns, I get totals of, umm…17 and 22. So we need to replace a number in the left column with a bigger number from the right column…what about swapping 5 and 7? It gives 19 and 20, closer but not right …
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Khali's approach is explorative with trial and error. This is a good start! Khalil engages with the task. However, he may need a hint to observe that some of the pairs would not work and they can be omitted. We see this in Mel's approach.

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Mel:	Hold on, if I add up the columns, I get totals of, umm…17 and 22. So we need to replace a number in the left column with a bigger number from the right column…what about swapping 5 and 7? It gives 19 and 20, closer but not right …
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Mel's approach is also explorative with trials. However, there is an observation that not all the pairs should be tested. It seems that her approach is more strategic! It would be good such strategic approaches to be commended and shared with the class. A strategic approach may have the potential to lead towards a general observation about the numbers in the two columns, see Wendy's observation.

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Wendy's approach not only puts forward a conjecture that there is no solution (abductive reasoning). She also proposes a more general explanation: if the difference is odd number, we cannot identify a pair of numbers. An alternative argument that may emerge is that "if the total of numbers is odd, we cannot identify a pair of numbers". There is an opportunity here to extend the discussion to arguments such as: "when the total of the two columns is odd (or even), the difference of the two columns is odd (or even)" or "how can we justify such argument?" or "if the total (or the difference) is even, can we identify a pair of numbers that would work?", etc.

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Callum represents the type of student who does not engage with an activity without a clear course of action. He does not see the point of the exploration, especially when there is a risk of not finding a solution. Callum's response aims to trigger a discussion around:

- views of mathematics as set of predefined steps that always lead to one and correct response or
- affective issues of student reluctance to engage with explorative activities.

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MathTASK: Theoretical Perspective-Theoretical constructs

- Discursive perspective that draws on the *theory of commognition* (Sfard, 2008)
- Mathematical and pedagogical discourses are established in communities and reflect the practices of those communities

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When we asked a mathematics teacher if they would “accept a graph-based argument as proof”, he replied:

“Mathematically, in the classroom, I would welcome it at lesson-level and I would analyse it and praise it, but not in a test”.

Asked to elaborate, he said:

“Through [the graph-based argument] I would try to lead the discussion towards a normal proof...with the definition, the slope, the derivative, etc.”

Asked to justify he said:

“This is what we, mathematicians, have learnt so far. To ask for precision. ... we have this axiomatic principle in our minds. ... And this is what is required in the exams. And we are supposed to prepare the students for the exams.

(Biza, Nardi & Zachariades, 2009, p. 34)

MathTASK:
Classification of mathematics teachers' arguments

- **A priori warrant:** resorting to a mathematical theorem or definition (a priori– epistemological) or resorting to a pedagogical principle (a priori– pedagogical);
- **Institutional warrant:** a justification of a pedagogical choice on the grounds of it being recommended or required in a textbook (institutional– curricular) or on the grounds that it reflects the standard practices of the mathematics community (institutional– epistemological);
- **Empirical warrant:** the citation of a frequent occurrence in the classroom (according to the arguer's teaching experiences, empirical–professional) or resorting to personal learning experiences in mathematics (empirical– personal);
- **Evaluative warrant:** a justification of a pedagogical choice on the grounds of a personally held view, value or belief.

(Nardi et al. 2012, pp. 160-161)

Toulmin's (1958) model of informal arguments and Freeman's (2005) classification of warrants

MathTASK: Theoretical Perspective-Theoretical constructs

- Discursive perspective that draws on the *theory of commognition* (Sfard, 2008)
- Mathematical and pedagogical discourses are established in communities and reflect the practices of those communities
- Mathematics and mathematics education are distinctive discourses and learning of mathematics and mathematics education is a communication act within these discourses.
- Students/teachers transform what they know about mathematics and about mathematics education theory into discursive objects that are used to describe teaching and learning.
- The discursive activity of *reification* (Sfard, 2008)

MathTASK:
Typology of four characteristics

Consistency: how consistent is a response in the way it conveys the link between the respondent's stated pedagogical priorities and their intended actions?

Specificity: how contextualized and specific is a response to the teaching situation under consideration?

Reification of pedagogical discourse (RPD): how reified is the pedagogical discourse, the theories and findings from research into the teaching and learning of mathematics – that respondents have become familiar with (e.g. through a mathematics education course) – in their responses?

Reification of mathematical discourse (RMD): how reified is the mathematical discourse – that respondents are familiar with (e.g. through prior mathematical studies) – in their responses?

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- The discursive activity of *reification* (Sfard, 2008)
- Teachers' capacity in identifying, interpreting and acting upon students' needs – **Teachers' capacity in noticing**

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The *Simplification* mathtask

In a Year 10 middle attaining class you have invited the students to solve the following problem:

"When p = 2.8 and c = 1.2, calculate the expression: $3c^2+5p-3c(c-2)-4p$."

After working on the problem for some time you invite the students to share their solution with the class. The dialogue below follows:

YOU: Ok, let's see what we can do with this question. Who wants to share their answer with me?

Student A and Student B raise their hands at the same time.

YOU: Student A?

STUDENT A: I found 10.

YOU: How did you find 10?

STUDENT A: I substituted the values 2.8 and 1.2 in the expression. It took me ages.

YOU: Thank you Student A! [To the class] Does everyone agree?

STUDENT B: I have the same answer but I did it so much quicker.

YOU: Go on…

STUDENT B: I worked out the expression before substituting the numbers and I ended up with a much simpler expression: $p+6c$. Then I substituted the values 2.8 and 1.2 and I found 10, easy!

STUDENT A: I like the way I did it; I don't like simplifying.

STUDENT B: My solution is brilliant, yours takes ages. You cannot work with letters because you are thick [Some students are giggling] … what can I expect from you anyway? [Some students are laughing].

You heard what Student B said …

Questions:

a. How are you going to respond to Student A, to Student B and to the whole class?

b. What do you think are the issues in this situation?

c. How are you going to deal with these issues in the future?

The **classroom situation context:** *Year 10* (age 15-16 years old), *middle attaining* class (students in the England are grouped according to the attainment to bottom, middle and top sets.

Mathematical problem: Calculate an algebraic expression for specific values of p and c; common mathematical problem in the Year 10 English curriculum; research reports students' difficulties in their work with variables in Algebra (e.g. Arcavi, 2005) and tensions between instrumental and relational understanding (Skemp, 1976)

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A list of **questions** that invite participants to engage with and reflect upon the situation

Classroom situation:
Inspired by the challenges of in-service teachers experience when they face the complexity of the classroom as this has been reported in practice and research (e.g. tensions between managing the classroom and simultaneously attending students' ideas), see more in Biza el al. (2015).

In relation to learning and teaching issues

- Different approaches by two students: substituting the values and then doing the calculations or simplifying the algebraic expression and then substituting the values;
- both solutions are correct;
- different qualities between solutions, proficiency in important algebraic skill vs working on extensive arithmetic operations; and,
- acceptance and appreciation of different solution.

Classroom management issues

- Tension between students;
- students' mutual respect;
- sharing and critiquing ideas in a classroom; and,
- dealing with situations of misbehaviour.

Teacher responses to the *Simplification* mathtask

- **21 pre-service teachers** following a mathematics initial teacher education course in a UK university.
- All respondents addressed student B's ill-behaved reaction to student A – <u>A clear priority to establish certain social norms in the classroom</u>

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- All teachers addressed **student B's ill-behaved reaction to student A** – <u>A clear priority to establish certain social norms in the classroom</u>
- The responses varied regarding the emphasis put on **the mathematical aspects of the incident**, especially in relation to the value of the two approaches to the problem and the potential difficulties of student A with simplification – An opportunity for teachers to reflect on the <u>sociomathematical norms they would establish in their classroom</u>
- **11 out of 21 responses** include evidence of at least one of the following:
    - the two solutions are not of equal value;
    - student A has difficulties with algebra;
    - the response addresses student A's difficulties.
- The **remaining 10 responses** consider the two solutions of equal value and, although three mention that student B's solution can be seen as quicker, they do not address student A's difficulties with algebra.

Biza, Nardi & Joel (2015)

MathTASK: Current work and ways forward

- We have been using **situation specific tasks (mathtasks)** in **research,** in **teaching** and in **teacher education**
  - **Undergraduate** and **master's** level courses on mathematics education
  - **Initial teacher education** courses (e.g. reflection on teaching practices – PRACTICUM)
  - **In-service teacher professional development** courses
- Classroom situation are **created by** researchers or teachers or co-created by researchers and teachers
- Classroom situation are used now for **students' activities** (as well as for teacher reflection)
- Mathematical problems and classroom situations are designed for (or adapted) to address **educational levels/contexts/specific foci**.

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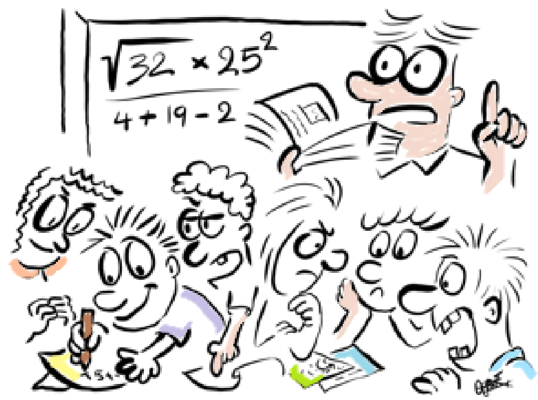
Many Thanks! Wielkie dzięki!



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MathTASK Website: https://www.uea.ac.uk/groups-and-centres/a-z/mathtask

MathTASK Video: https://youtu.be/gt0HZBfBBGI

References from the MathTASK programme

Biza, I. & Nardi, E. (2023). Challenging undergraduate students' mathematical and pedagogical discourses through MathTASK activities. In R. Leikin (Ed.), *Mathematical Challenges for All*, , (pp. 343-363). Springer. https://doi.org/10.1007/978-3-031-18868-8

Nardi, E. & Biza, I. (2022). Teaching Mathematics Education to Mathematics and Education Undergraduates. In R. Biehler, G. Gueudet, M. Liebendörfer, C. Rasmussen & C. Winsløw (Eds.*), Practice-Oriented Research in Tertiary Mathematics Education: New Directions*, (pp. 311-327). Springer. https://doi.org/10.1007/978-3-031-14175-1

Biza, I., Kayali, L., Moustapha-Corrêa, B., Nardi, E. & Thoma, A. (2021). Afinando o Foco em Matemática: Desenho, Implementação e Avaliação de Atividades MathTASK para a Formação de Professores de Matemática. [Sharpening the focus on mathematics: Designing, implementing and evaluating MathTASK activities for the preparation of mathematics teachers.] Perspectivas da Educação Matemática, 14(35), 1-41. Original version available at: https://periodicos.ufms.br/index.php/pedmat/article/view/13407/9353. English version available at: https://ueaeprints.uea.ac.uk/id/eprint/80974/2/Accepted_Manuscript.pdf.

Biza, I. & Nardi, E. (2019). Scripting the experience of mathematics teaching: The value of student teacher participation in identifying and reflecting on critical classroom incidents *International Journal for Lesson and Learning Studies, 9*(1), 43-56. Publisher page. UEA Repository

Biza, I., Nardi, E., & Zachariades, T. (2018). Competences of mathematics teachers in diagnosing teaching situations and offering feedback to students: Specificity, consistency and reification of pedagogical and mathematical discourses. In T. Leuders, J. Leuders, & K. Philipp (Eds.), *Diagnostic Competence of Mathematics Teachers. Unpacking a complex construct in teacher education and teacher practice,* (pp. 55-78). New York: Springer. Final published version.

Nardi, E., Healy, L., Biza, I., & Fernandes, S.H.A.A. (2018). 'Feeling' the mathematics of disabled learners: Supporting teachers towards attuning and resignifying in inclusive mathematics classrooms. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners,* (pp. 147-170). SENSE Publications. Final published version.

Biza, I., Nardi, E., & Joel, G. (2015). Balancing classroom management with mathematical learning: Using practice-based task design in mathematics teacher education. *Mathematics Teacher Education and Development, 17*(2), 182-198. Publisher Website UEA Repository

Nardi, E., Biza, I., & Zachariades, T. (2012). 'Warrant' revisited: Integrating mathematics teachers' pedagogical and epistemological considerations into Toulmin's model for argumentation. *Educational Studies in Mathematics, 79(2),* 157-173. Available at: https://ueaeprints.uea.ac.uk/36867/1/2012_Nardi_Biza_Zachariades_ESM_79_2_157_73_Repository.pdf

Biza, I., Nardi, E., & Zachariades, T. (2009). Teacher beliefs and the didactic contract on visualisation. *For the Learning of Mathematics, 29*(3), 31-36.

Biza, I., Nardi, E., & Zachariades, T. (2007). Using tasks to explore teacher knowledge in situation-specific contexts. *Journal of Mathematics Teacher Education, 10,* 301-309. Available at: https://ueaeprints.uea.ac.uk/36871/1/2007_JMTE_Biza_Nardi_Zachariades_10_301_309_Repository.pdf

More about the MathTASK Programme can be found in the MathTASK Website: https://www.uea.ac.uk/groups-and-centres/a-z/mathtask and the MathTASK publication page.