



Dipartimento  
Matematica

UNIVERSITÀ  
DI TORINO

# Lesson Study transposition to Italy

## Cultural challenges

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# About me



Doctoral school

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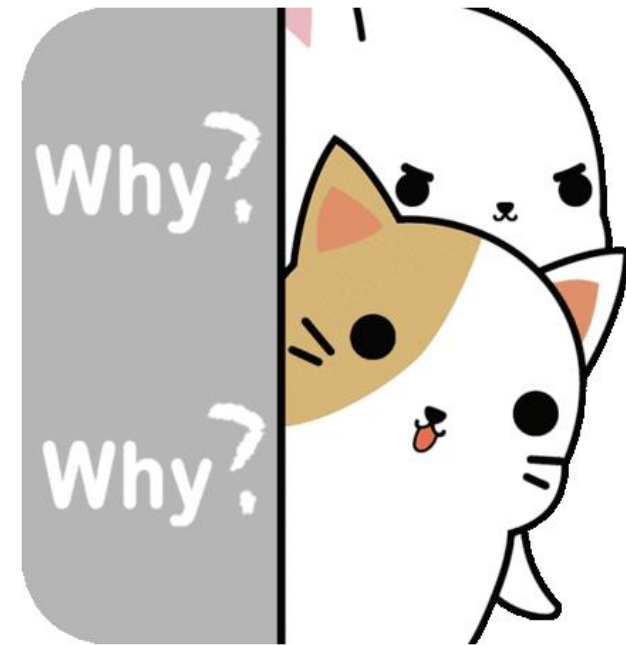
Research focus: teachers' professional development (PD) in collaborative contexts

# Presentation plan

- Lesson Study: what is it, and why do we study it?
- The role of culture in professional development
- The Japanese and Italian Institutional contexts
- The experiment with prospective teachers
- Some results
- Future perspectives and Discussion

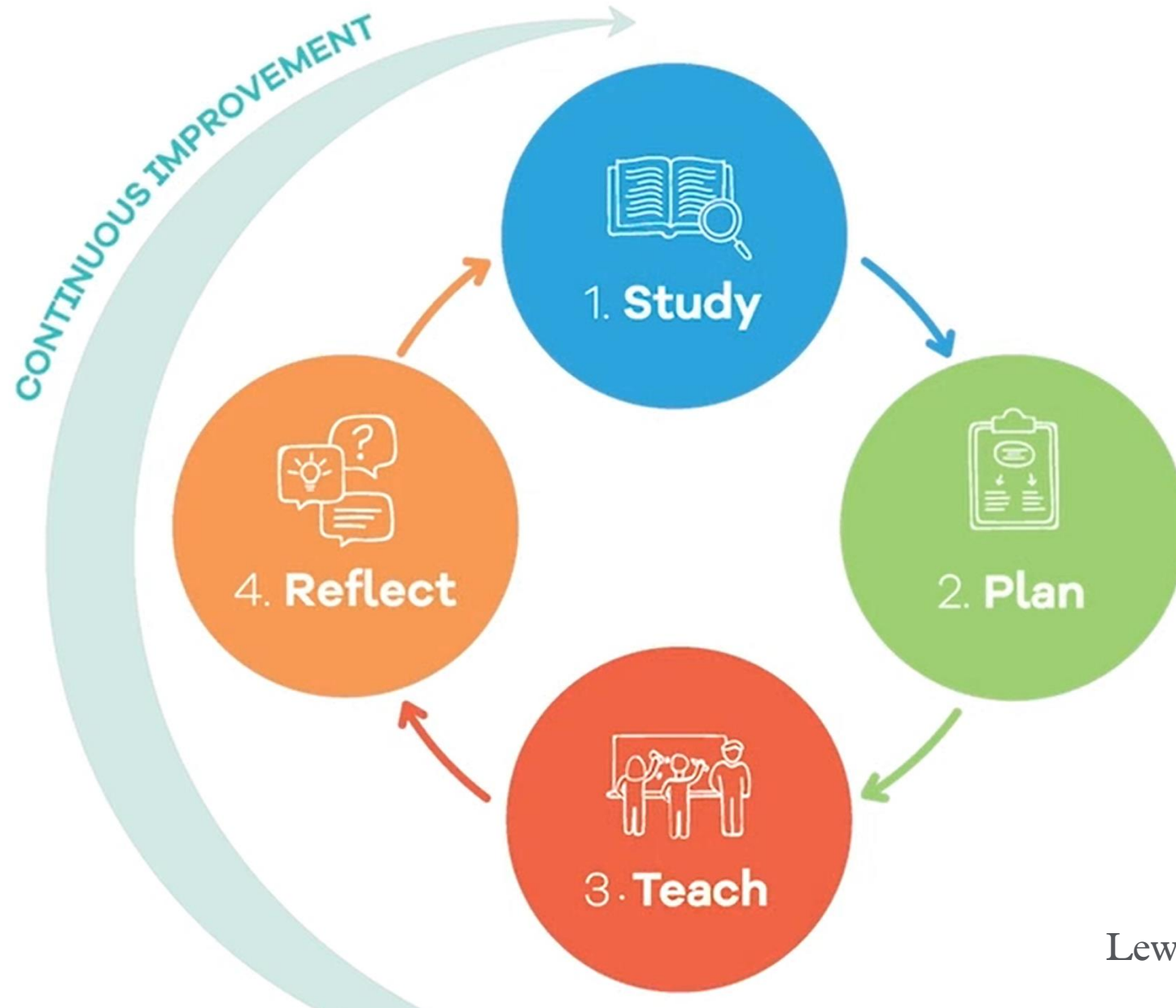


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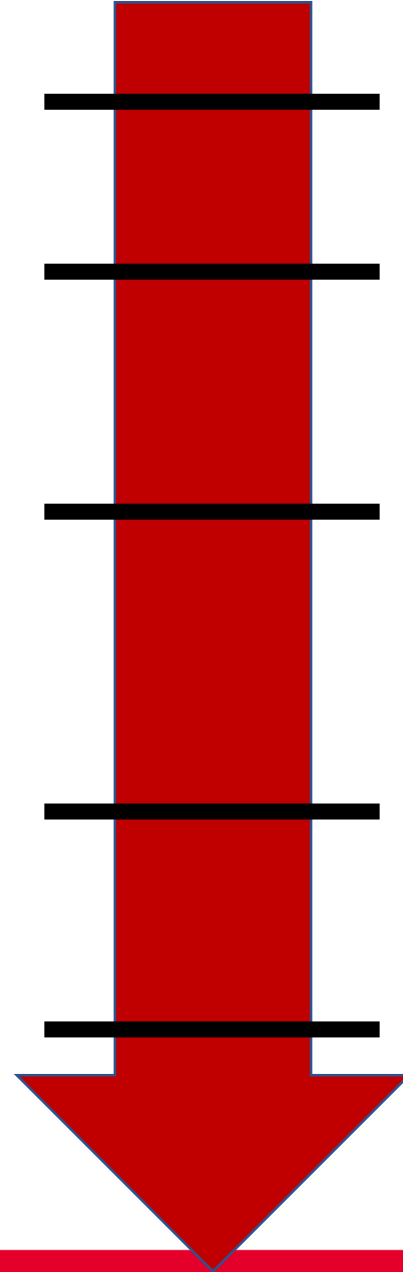
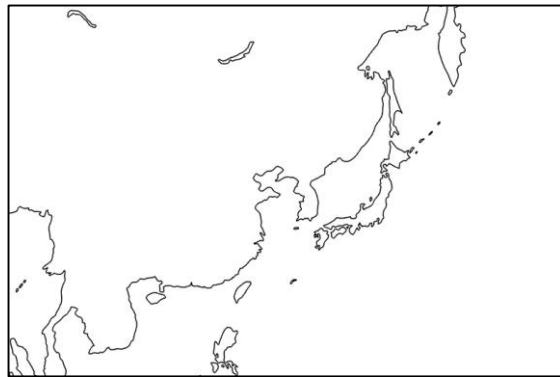


# What is Lesson Study



Lewis et al. (2019)

# A brief history of Lesson Study



**1872** Emperor Mutsuhito's **reform** of the schooling system

**1872 - 1880** Opening of **teachers' schools**

**1880** A book on **Pestalozzi's peer-education methods** inspires the first spontaneous Lesson Studies

**1900** LS migrates to China and Korea

**1999** LS reaches the world

Isoda et al. (2007)

# Lots of Western sources on LS

- Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Free Press.
- Fernandez, C., & Yoshida, M. (2004). *Lesson Study: A Japanese Approach To Improving Mathematics Teaching and Learning*. Routledge
- White, A. L., & Lim, C. S. (2008). Lesson study in Asia Pacific classrooms: Local responses to a global movement. *ZDM*, 40(6), 915–925.
- Winsløw, C. (2011). A Comparative Perspective on Teacher Collaboration: The Cases of Lesson Study in Japan and of Multidisciplinary Teaching in Denmark. In *From Text to 'Lived' Resources* (pp. 291–304). Springer Netherlands.
- Demir, K., Sutton-Brown, C., & Czerniak, C. (2012). Constraints to Changing Pedagogical Practices in Higher Education: An example from Japanese lesson study. *International Journal of Science Education*, 34(11), 1709–1739.
- Dudley, P. (2014). *Lesson Study: A Handbook*.
- Ebaegu, M., & Stephens, M. (2014). Why Lesson Study Works in Japan: A Cultural Perspective. *Mathematics Education Research Group of Australasia*, 199–206.
- Groves, S., Doig, B., Vale, C., & Widjaja, W. (2016). Critical factors in the adaptation and implementation of Japanese Lesson Study in the Australian context. *ZDM*, 48(4), 501–512.
- Lewis, C. (2016). How does lesson study improve mathematics instruction? *ZDM*, 48(4), 571–580.
- Stigler, J., & Hiebert, J. (2016). Lesson study, improvement, and the importing of cultural routines. *ZDM*, 48(4), 581–587.
- Warwick, P., Vrikki, M., Vermunt, J. D., Mercer, N., & van Halem, N. (2016). Connecting observations of student and teacher learning: An examination of dialogic processes in Lesson Study discussions in mathematics. *ZDM*, 48(4), 555–569.

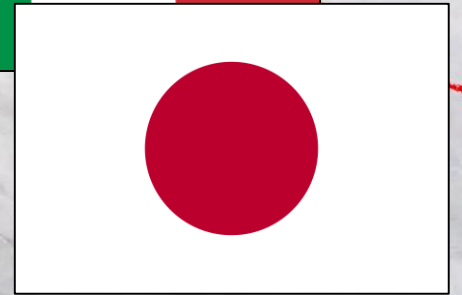
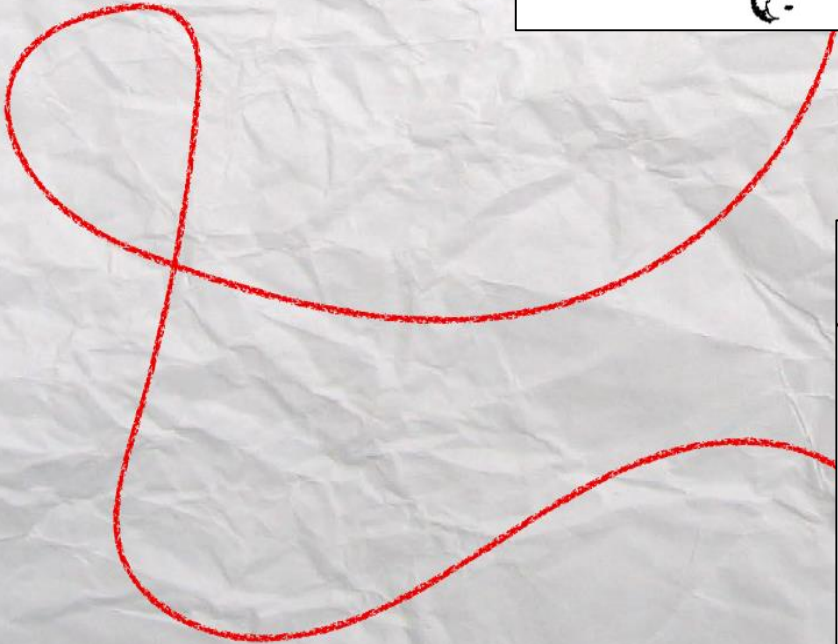


# Different perspectives

From a «western» point of view...

Lesson Study lacks its own 'original' theoretical characterization.

This is likely due to its roots in the Eastern culture - far from the Western **need** for theorizing (Yang, 2009).



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# Culture and Mathematics

## Values and mathematics

- Mathematics is shot through with values
- $V-E+F = 2$  (simple properties that apply generally)
- Prime numbers (small set of underlying generators)
- The Langlands programme (connection across difference)
- Values change over time and across cultures
  - Focus of some cultures on calculations (India)
  - Interest in visual demonstrations (China)
  - Physical forms of argument (Archimedes)

Nathalie Sinclair, February 24<sup>th</sup>, SoME (<https://youtu.be/PiVyirJS4hk>)



# Culture and professional development

“The role of culture [...] is an important aspect of mathematics education” (Presmeg, 2007, p. 1)

In an education system, **one expects culture to contribute to** the forms of acceptable pedagogy, social conventions governing teacher interactions, classroom practice, and **teacher professional development programs.**

Ebaeguin and Stephens (2014)

# What is «culture»?

“any aspect of the ideas, communications, or behaviours of a group of people which give them a distinctive identity and which is used to organise their internal sense of cohesion and membership” (Scollon & Scollon, 1995, p. 127)

“[t]he system of shared beliefs, values, customs, behaviours, and artefacts that the members of society use to cope with their world and with one another, and that are transmitted [...] through learning” (Bates & Plog, in Freimuth, 2006, p. 2)

# What is «culture»?

There is no shared definition of *culture* (Spencer-Oatey, 2012), but it is often linked with the concept of society and organization.

We rely on “the notion of “practice” as a link between culture [...] and the larger cultural contexts” (Hatano & Inagaki, 1998, p. 80).

# Culture as institutions (ADT)

To deal with this cultural aspect of PD we adopt the **institutional perspective** proposed within the Anthropological Theory of the Didactic (ATD). **Institution** is “any created reality of which people can be members (permanent or temporary)” (Chevallard & Bosch, 2020, p. xxxi).

Peoples’ practices and knowledge are influenced or shaped by diverse elements, called **conditions and constraints**, prevailing in the **institution** to which they belong.

One of the biggest contributions of ATD was to focus, in the case of mathematics teaching, on not only those in the classroom **but also those beyond**, such as cultural elements related to school, society, civilization, etc. (Bosch & Gascón, 2006).

The institutional perspective of ATD allows investigating the **cultural aspect of human practices**.



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# Japanese school context



- **6-3-3 system**, mandatory until 9th grade
- **Non-inclusive** classrooms
- School day **starts at 08:50 and ends at 16:00** (teachers from 08:00 to 17:00)
- PD is **not mandatory**, but almost every teacher participates
- PD mostly happens **during** school hours
- PD is often organised by schools
- PD is financed
- Classrooms are **open** to observers
- Teachers have **shared spaces** for discussion

# Italian school context



- **5-3-5 system**, mandatory until 10th grade
- **Inclusive** classrooms
- School day can be from **08:00 to 17:00** (students and teachers stay in school only/mainly for the lessons)
- PD is **mandatory** but not regulated
- PD takes place during **non-school** hours
- Teachers voluntarily adhere to PD
- **Little** financial support
- Classrooms are «**teacher's private space**»
- **No** shared spaces for discussion

Minisola and Manolino (2022)

# About curriculum



“The mathematics curriculum consists of three parts: overall objectives for the level, objectives and content for each grade, and **syllabus design**.

Methods and materials **are specified to some extent**, as well as in **the construction of teaching plans** and remarks on content.”

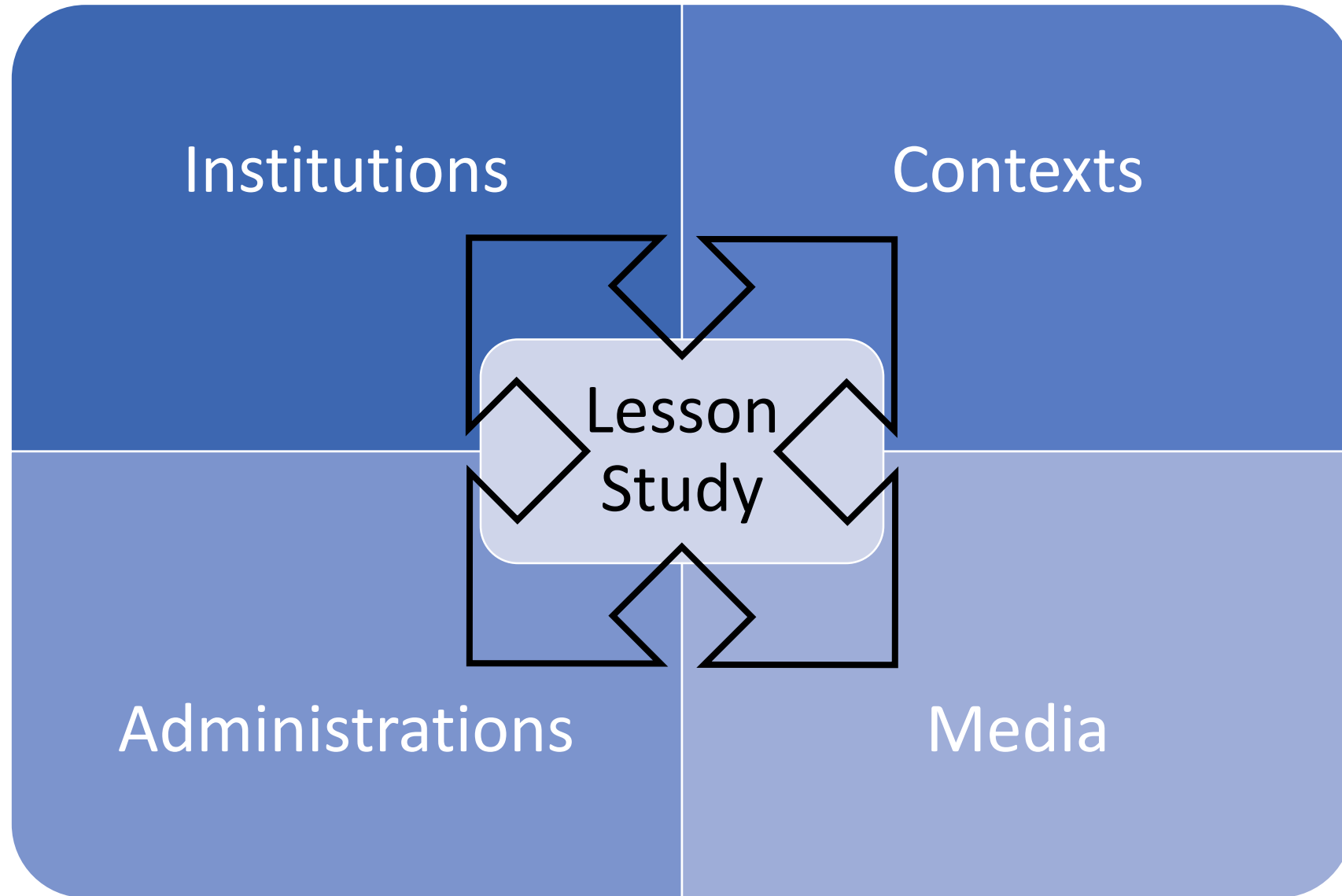


The Ministry of Education publishes the Indicazioni Nazionali, “a framework for schools in the design and implementation of local primary and lower secondary curricula. Schools **are free to determine content and methods of instruction autonomously**, provided they are consistent with the learning objectives established by the Indicazioni.”

«Teaching freedom» is a **constitutional right**.

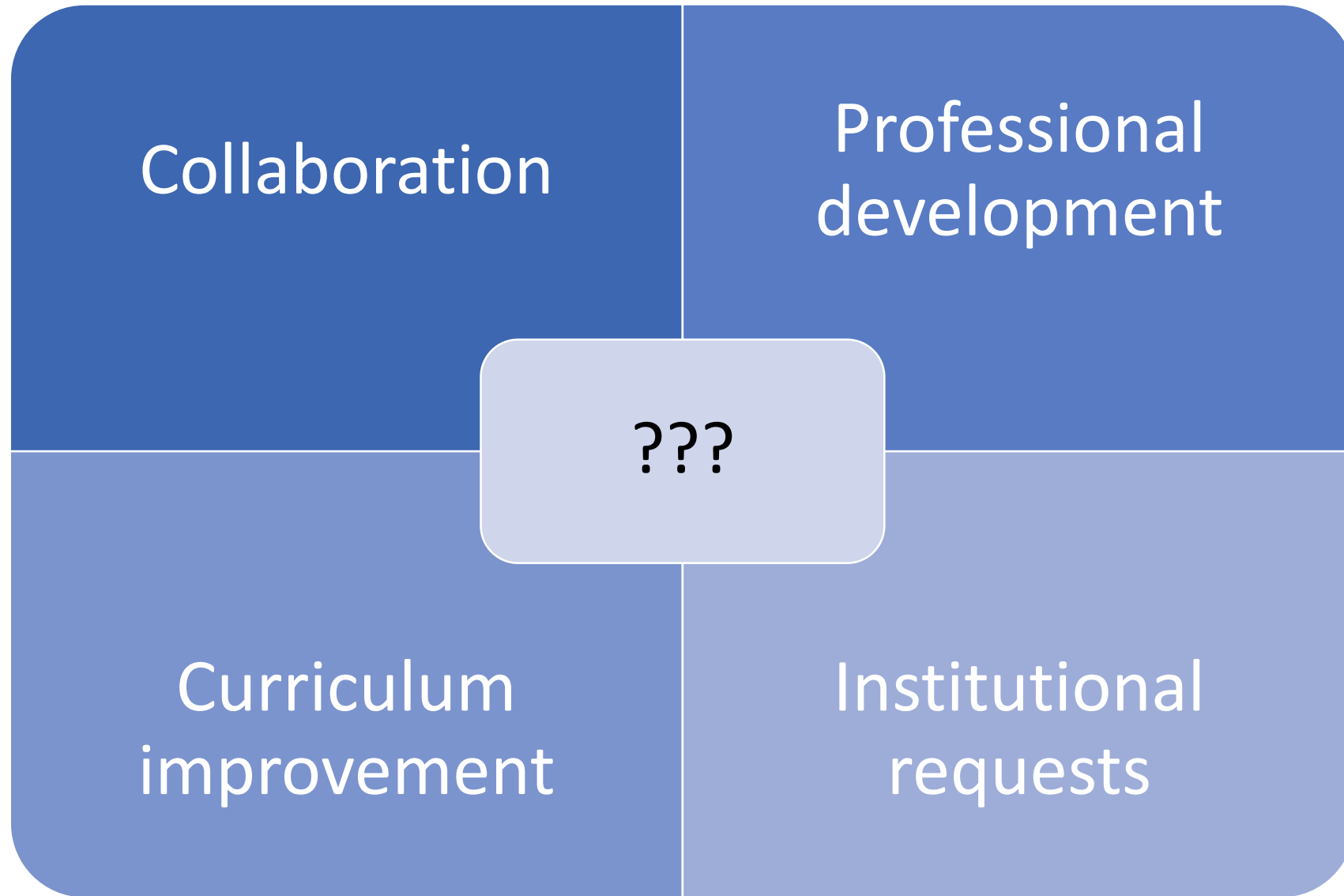
<https://timssandpirls.bc.edu/timss2015/encyclopedia/>

# Japanese «sinergies»

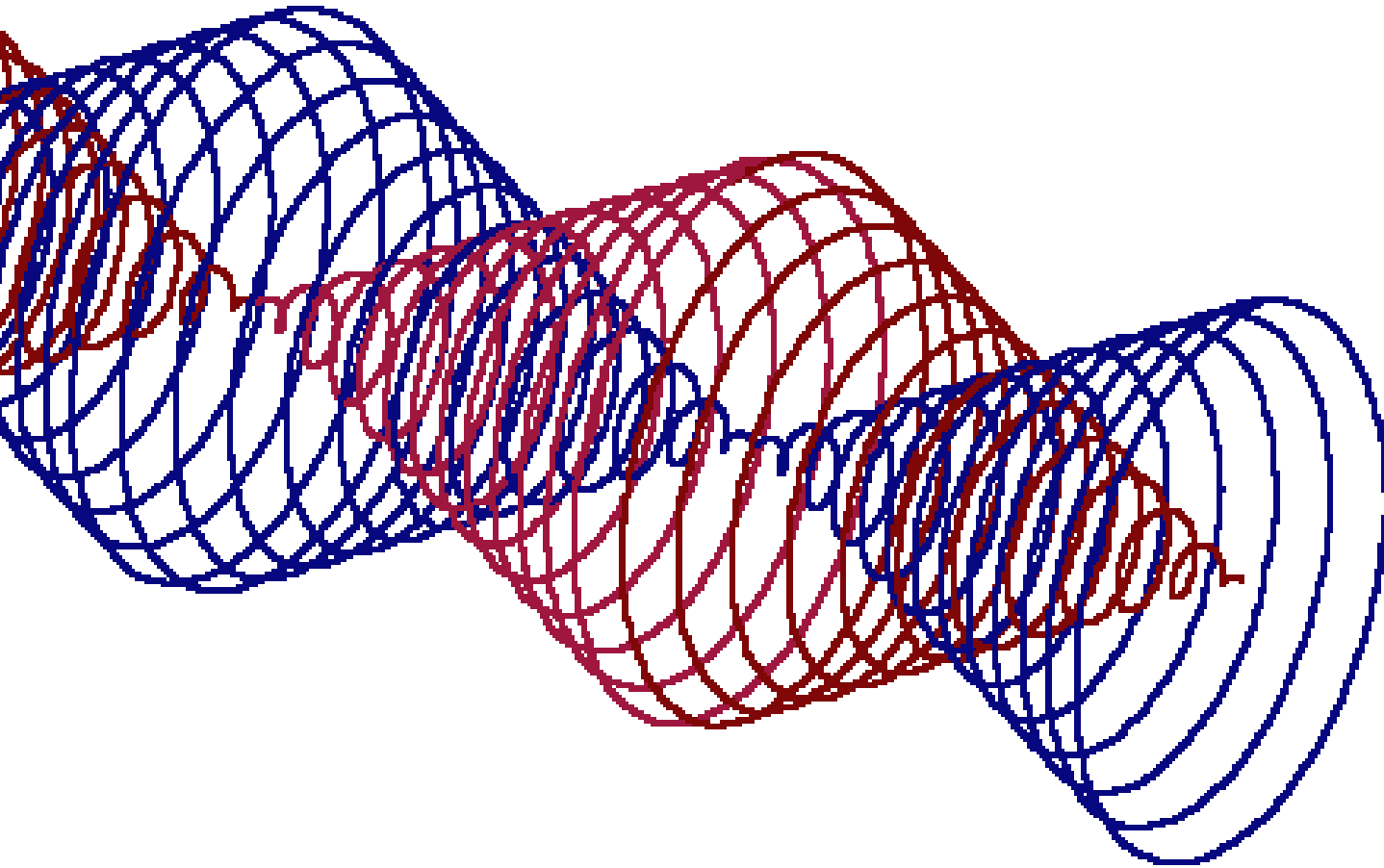


Miyakawa and Winsløw (2019)

# Why could LS help Italy



# Tackling the problem



Study of Japanese and Italian  
culture and institutions



Our understanding of LS as  
researchers



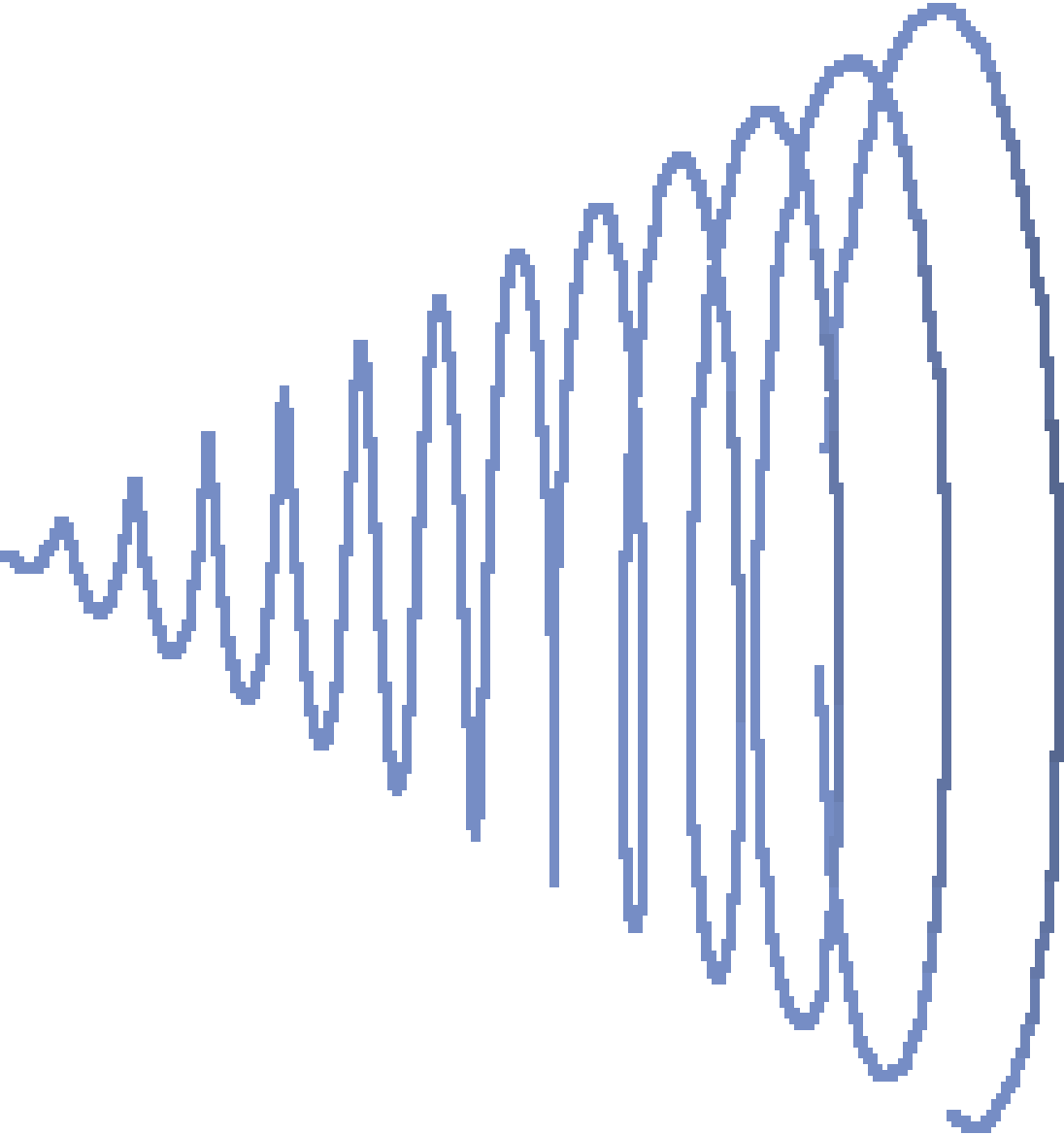
Experiments with prospective  
and practicing teachers

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# The experimental setting



And, of course, the researchers as «educators»

- 29 teachers in pre-service training (UniTo students)
- 22-24 yo
- **No real classroom experience**
- Enrolled in the course *Elementary Mathematics from an Advanced Standpoint*, held from October to December 2018
- Topic of the course: **continued fractions**

# Introduction to Lesson Study

## From the beginning to the present day

- **1872** Emperor Mutsuhito (Meiji Restoration) **reforms** the education system.
- **1872 - 1880** First **normal schools** open (University of Tsukuba)
- **1880** A book about **Pestalozzi** leads to the first spontaneous Lesson Study experiences.
- **1920** From Dewey's theories comes teaching by peer-learning and open problem solving
- **1945--** Lesson Study arrives in secondary schools

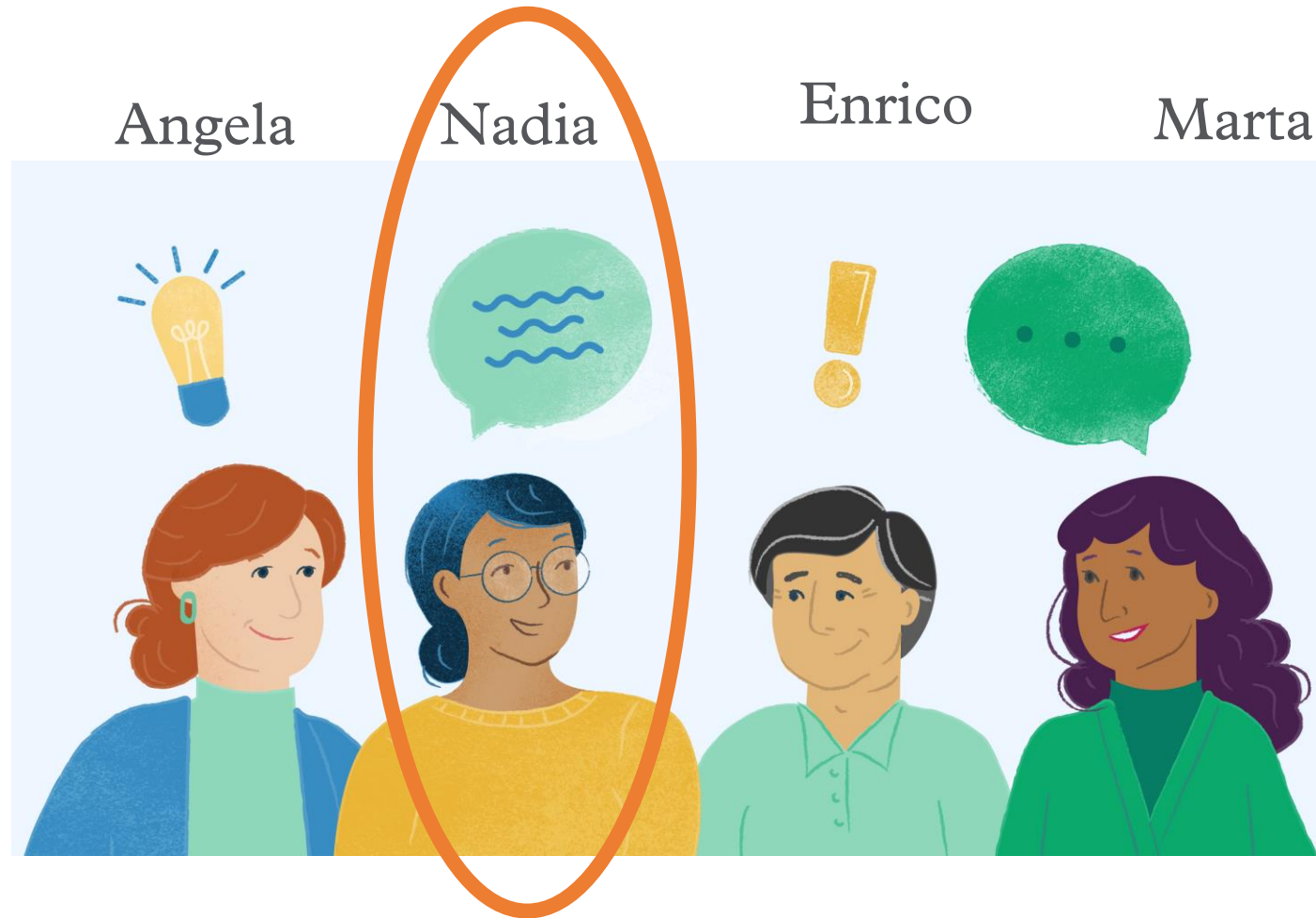
## The phases of the (J)Lesson Study

1. **Defining goals** Long-term educational goals are decided in accordance with the national programme and the school's mission
2. **Lesson Planning** Drawing up of the detailed Lesson Plan
3. **Research lesson** Teacher or expert teaches, other participants observe students' working in the class
4. **Discussion** Discussion is always based on students' reactions, the quality of the lesson itself is not important
5. **Reflection** Teachers metabolize what they have learned and produce a written text that remains for historical memory

# The task for the teachers

- *Planning phase*: study and design a teaching activity on continued fractions, and write an **activity report** with a **lesson plan** for a 20-minute mock lesson.
- *Implementing phase*: teach and observe this mock lesson in front of their peers and the educators.
- *Reflecting phase*: collaboratively discuss the efficacy of the lesson and individually reflect on the discussion.

# Group 1



# Activity report of Group 1

## Progettazione attività 2

**Titolo:** "Frazioni matryoska".

**Anno di riferimento:** terzo anno di scuola secondaria di primo grado.

**Contesto di apprendimento:** intra-matematico (numeri razionali, figure geometriche).

**Prerequisiti:** conoscenza dei numeri razionali, operazioni tra numeri razionali, figure geometriche di base (rettangolo, quadrato).

**Nodi concettuali:** suddivisione grafica, frazioni, frazioni continue.

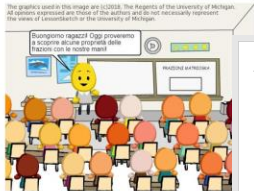
**Riferimento alle indicazioni nazionali per la scuola secondaria di primo grado:**

- Eseguire semplici calcoli con numeri razionali usando metodi e strumenti diversi;
- Risolvere problemi usando proprietà geometriche delle figure ricorrendo a modelli materiali e a semplici deduzioni e ad opportuni strumenti di rappresentazione (riga, squadra, compasso e, eventualmente, software di geometria);
- Esprimere verbalmente in modo corretto i ragionamenti e le argomentazioni;
- Schematizzare anche in modi diversi la situazione di un problema, allo scopo di elaborare in modo adeguato una possibile procedura risolutiva;
- Esperire chiaramente un procedimento risolutivo, evidenziando le azioni da compiere e il loro collegamento.

### Fase 1

Durata: circa 20 minuti

- Presentazione dell'attività agli alunni, con il supporto della lavagna: disegno + consegna. Spazio per domande veloci su dubbi degli alunni.



- Consegna delle schede ad ogni studente.



In ogni scheda è disegnato un rettangolo di lati noti, che dovrà essere suddiviso dagli studenti nel maggior numero possibile di quadrati; inoltre, è richiesto il completamento di una sequenza di "conti".

La scheda potrebbe presentarsi nel seguente modo:

**Suddividi il rettangolo in quanti più quadrati (di lato 16) possibile, utilizzando la suddivisione del rettangolo ausiliario.**

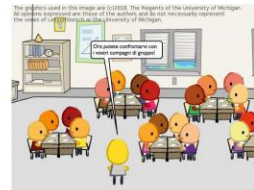
**Completa inserendo i numeri opportuni nei quadratini bianchi**

$$\begin{array}{r} 45 & 16 + & + 13 \\ \hline & 16 & \\ \hline & & 13 \\ & + & \\ & 16 & 16 \\ \hline & & 13 \\ & = & 16 \end{array}$$

### Fase 2

Durata: 25-30 minuti

- Suddivisione della classe in gruppi eterogenei da 3 studenti: in questo caso, siccome ciascuno studente ha già apprezzato l'argomento da solo, non si correrà il solito rischio dei gruppi eterogenei, ovvero che i ragazzi più intraprendenti lavorino da soli imponendosi su quelli più timidi e incerti.



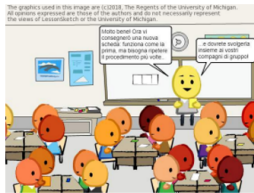
- Gli studenti di ciascun gruppo hanno 5-10 minuti per confrontarsi su quanto appena fatto (nella fase 1). L'insegnante gira tra i gruppi per verificare che ciascuno di essi sia sulla buona strada; se si accorge che uno o più gruppi hanno sbagliato la risoluzione della scheda 1, può chiamare alla lavagna uno studente (che abbia svolto correttamente la consegna) in modo che, grazie al confronto e alle domande, tutti possano capire lo svolgimento corretto della scheda.

- Si fornisce a ciascun gruppo una seconda scheda, nella quale si chiede di iterare il procedimento della scheda 1, ma su frazioni/rettangoli diversi (a seconda della reattività del gruppo).



tutti possano capire lo svolgimento corretto della scheda.

- Si fornisce a ciascun gruppo una seconda scheda, nella quale si chiede di iterare il procedimento della scheda 1, ma su frazioni/rettangoli diversi (a seconda della reattività del gruppo).



**Suddividi il nuovo rettangolo di lati 16 e 13 in quanti più quadrati (di lato 13) possibile, poi ridisegna il rettangolo di partenza e ripeti il procedimento per ogni nuovo rettangolo che trovi.**

16 16 13

45

**Completa inserendo i numeri opportuni nei quadratini bianchi.**

$$\begin{array}{r} 45 \\ 16 = 16 + 16 + 13 \\ \hline 16 = \\ = 32 + 13 \\ = 2 + 13 \\ = 2 + 1 \\ = 2 + \frac{1}{13} \\ = 2 + 1 \\ + 13 \\ = 2 + 1 \\ + 1 \\ = 2 + 1 \\ 13/ \end{array}$$

*F'in quanto potete andare avanti graficamente? F'in quanto continuate i conti?*

### Fase 3

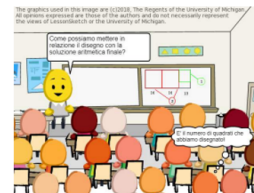
Durata: circa 20 minuti

- L'insegnante pone delle domande e se ne discute a livello di classe. Le domande possono essere:
  - Come avete risposto all'ultima domanda: "Fin quando andate avanti?"
  - Come si può collegare il disegno alla soluzione aritmetica finale?

$$\begin{array}{r} 45 \\ 16 = 2 + 1 \\ \hline 1 + 1 \\ 4 + \frac{1}{13} \end{array}$$

L'insegnante introduce la notazione  $45_{16} = [2; 1, 4, 3]$  come semplificazione della scrittura precedente; e inizia ad utilizzare il termine *frazione continua*.

- Spazio alle domande degli allievi, come spunti per poi formalizzare quanto hanno appena visto.



### Fase 4

- L'insegnante fa notare agli studenti che hanno appena scritto una frazione sotto forma di somma di frazioni a matryoska.

- Fornisce poi altri esempi e mostra il caso particolare, in un'esposizione interattiva con gli studenti.

### Fase 5

Possibile formalizzazione simbolica (in base al livello, all'interessamento e alla stanchezza degli studenti)

**Metodologie:**

- Spiegazione frontale introduttiva e conclusiva.
- Lavoro individuale.

# Matryoshka fractions

## Grade 11

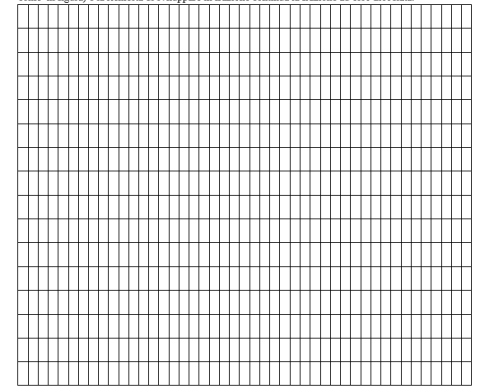
### 80+ minutes

- Attività di gruppo.
- Discussione generale.

**Suggerimenti per le verifiche:**

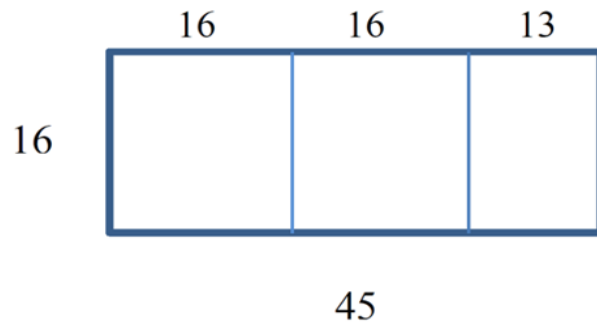
Alla fine di valutare le competenze acquisite dagli studenti nel corso dell'attività, si può proporre un esercizio nel quale ciascun alunno debba scrivere una frazione data in forma di frazione continua, con l'unico ausilio dell'approccio grafico, dunque senza l'utilizzo di calcoli.

Ad esempio, nella scheda per lo studente potrebbe esserci un rettangolo (suddiviso in quadratini unitari, come in figura) e la richiesta di sviluppare in frazione continua la frazione ad esso associata.



# Activity report of Group 1 (focus on)

*Divide the new rectangle [the rightmost one] with sides 16 and 13 in as many squares with side 13 as possible; then redraw the starting rectangle and repeat the process for each new rectangle you may find.*



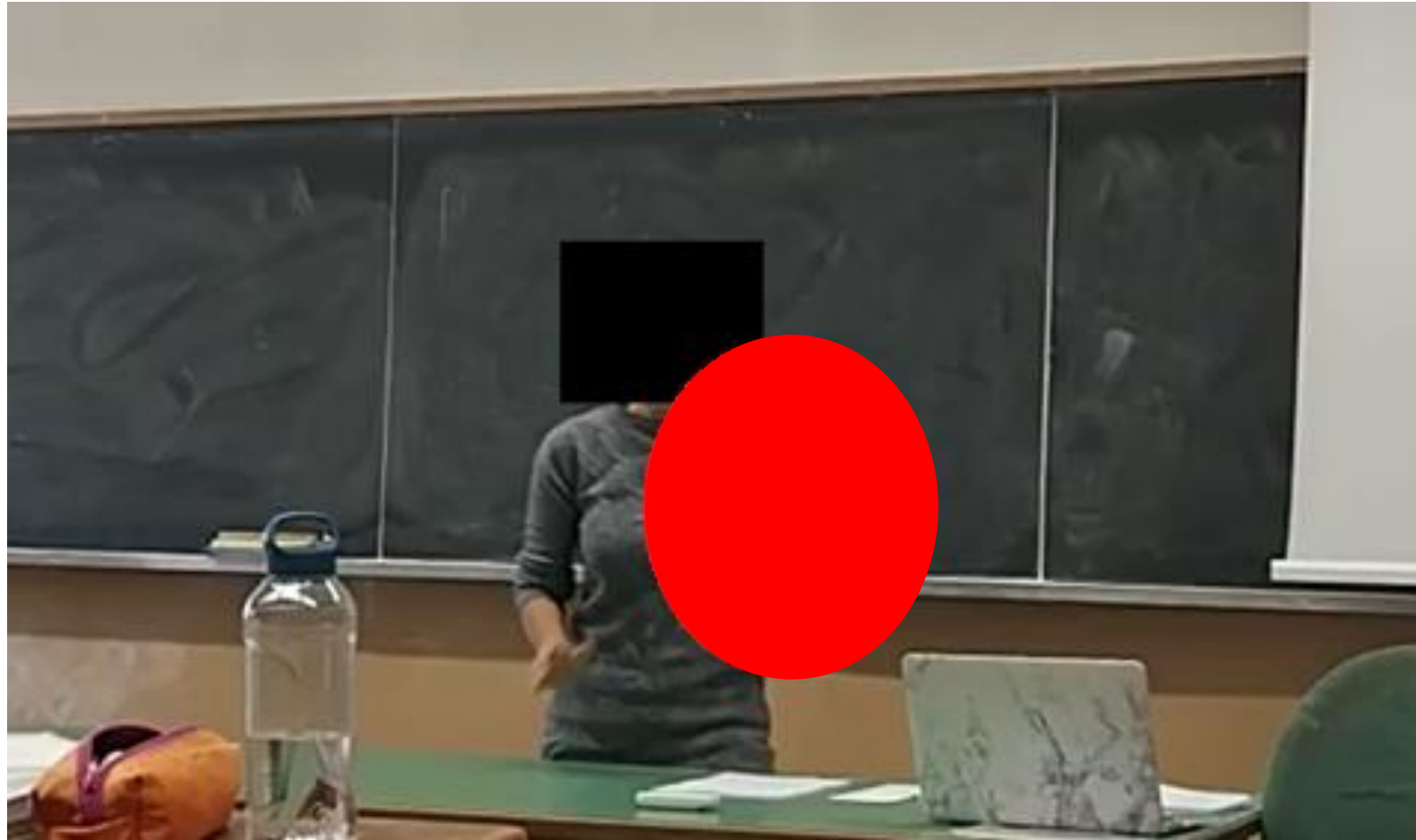
Fill in the empty squares with an appropriate number, and complete the process

$$\begin{aligned}\frac{45}{16} &= \frac{32}{16} + \frac{13}{16} = 2 + \frac{13}{16} = \\ &= 2 + \frac{1}{\frac{16}{13}} = 2 + \frac{1}{\frac{\square + \square}{13}} = 2 + \frac{1}{\square + \frac{\square}{13}} = \dots\end{aligned}$$

# Lesson Plan of Group 1



# Mock lesson of Group 1



Tools: blackboard, paper, drawings on the blackboard...



# Mock lesson of Group 1



Tools: blackboard, paper, drawings on the blackboard...

...matryoshka?

# Overview

<b>Group #</b>	<b>Learning context to approach continued fractions</b>	<b>Total time estimation</b>	<b>Lesson Plan</b>
1	Geometric representation of continued fractions	80+ minutes	No
2	Euclid's Algorithm in Geometry	Not present	No
3	Music and continued fractions	3 hours	No
4	Continued fractions and approximation	4 hours	No
5	Coding and visualizing Continued Fractions in MATLAB	Not present	No
6	GCD and continued fractions	1 hour	No
7	Continued Fractions and Golden spiral	Not present	No
8	Storytelling of Euclid's Algorithm	Not present	No

# From the survey...



- They were all convinced that they had produced a Lesson Plan
- Yet, the word «lesson» almost never appeared in their answer.
- When they referred to the mock lesson, they used the term «activity».

# We are not machines

Teacher  
(Nadia)

I personally chose to use the matryoshka to simplify the explanation of continued fractions, and to provide a clearer picture for the students. We did not discuss this among colleagues as the way of expressing the content [...] is very subjective and strongly depends on the teacher.

Teacher 1

I like that there is collaboration between teachers for the creation of a common project to be presented to students, but there must be flexibility in adapting the lesson to the class [...] according to the characteristics of the teacher (we are people and not machines).

# Introduction to LS (2)

Each group should have produced an activity report and a detailed lesson plan for the lesson to be presented. However, all groups only produced an activity report with eventually a general indication of the time [...].

How do we better communicate with teachers?

Has “what is meaningful in the world of LS research” the same meaning in the teachers’ community?

At the macro level, [...] we believe there is the need to establish a shared language *a priori*, in particular concerning the terms "lesson" and "activity".

# Introduction to LS (2)

## Phase 2: Detailed lesson plan

### What goes

- The activity is divided in sections
- Teaching strategies are different for each section
- Indication of when exercises or examples should be used

### What we can improve

- Little context description
- Time allocations are not specific
- No foreseeing of pupils reactions
- Little attention on the teacher's role
- Too many changes between different classroom settings in one section

# From the questionnaire...

Nadia	As a teacher, I realised that the planning was insufficient and that much space was left to my personal initiative. In retrospect, I believe that the time spent on planning should be much more. It is in fact a single lesson to be prepared really thoroughly, second by second.
	Planning is the cornerstone of a lesson [study] as it forces one to consider a multitude of aspects that are often left aside and improvised during a normal lesson.
Marta	Lesson [planning], as compared to activity design, is more linked to contextual aspects (the class to which the lesson is addressed and so on...)
Enrico	[...] we mixed up [activity] and lesson, our lesson was structured over two hours and not a single one-hour unit [...]. This may be because we are used to having longer times for individual lessons, so we mix activities and lessons.

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# Three «problematic features»

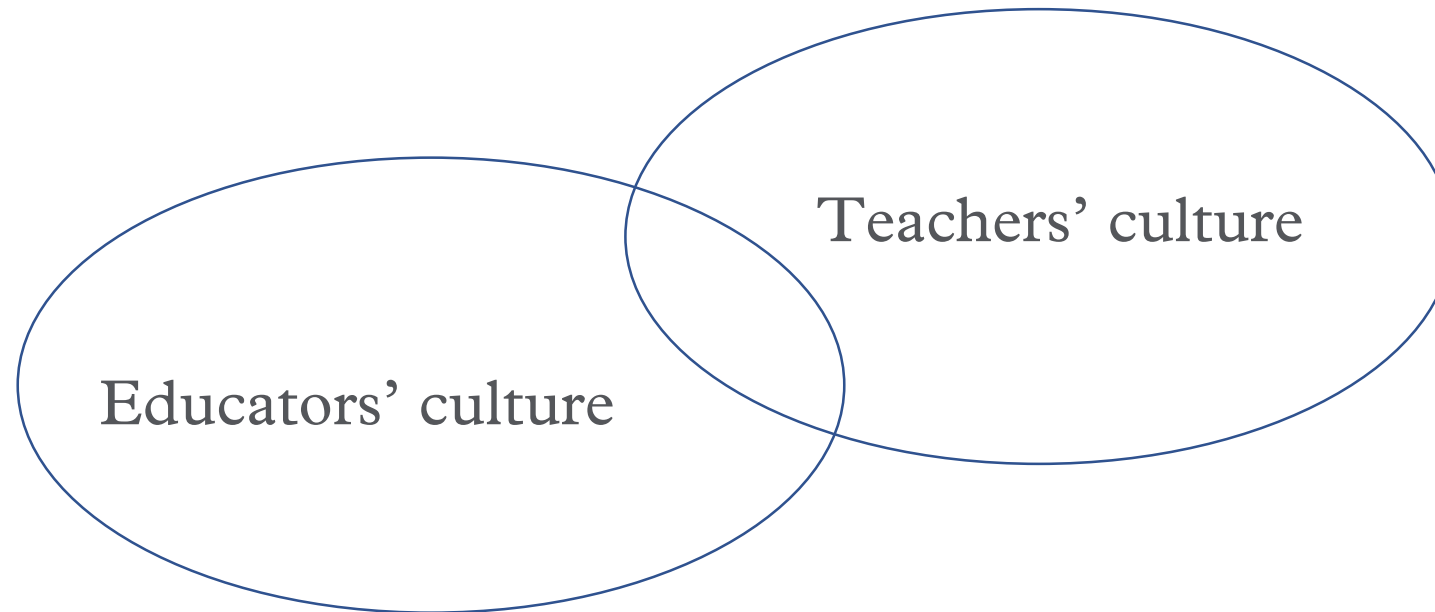
Coherently with other implementations of LS in Europe, a number of problematic features of LS emerged in this experiment.

The institutional differences between Italy and Japan determined three main problematic features:

- activity – lesson;
- lesson plan;
- time planning.

# Activity - Lesson

**Terminology** is a crucial cultural factor, as it could be a constraint that prevent proper implementation of LS (or other teachers' practices) in another context.



# Lesson Plan

The lesson plan is a specific tool of LS, whose importance was not easily understood in our experiment, since the activity report plays a similar role for Italian teachers.

First, it can allow teachers to be more aware of their educational choices and of the real-classroom context in which they are working when planning lessons.

Second, it can be a tool to share, collaborate and discuss the teaching with other colleagues.

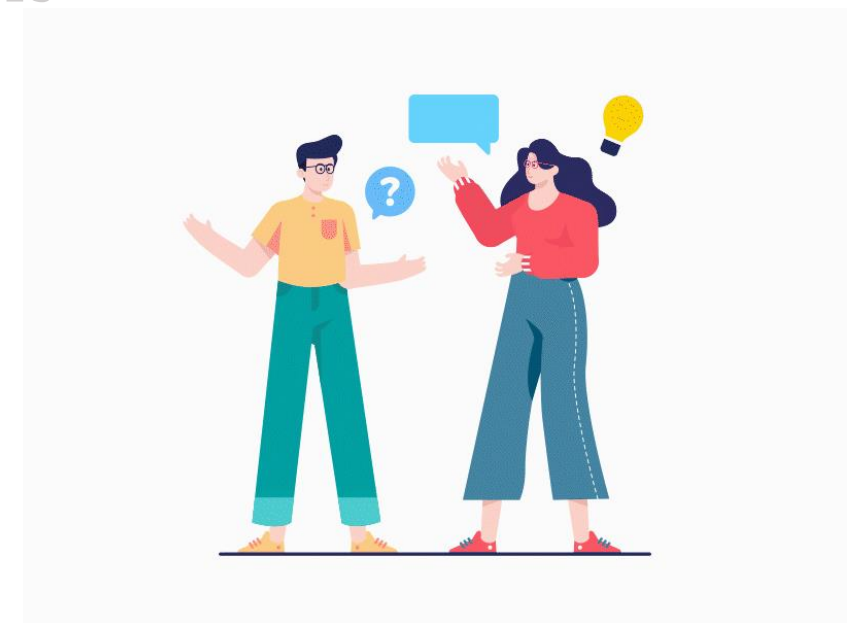
It is necessary to clarify these roles to the teachers, and emphasize that the lesson plan is not a rigid set of instructions.

# Time planning

The data suggest that it could be very difficult, for the Italian teachers, to time-plan a single lesson in detail, since they are not used to a meticulous time planning.

Interacting with LS and its tools may force teachers to explicit and pay attention to every moment of the lesson.

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# Interaction with the institutions

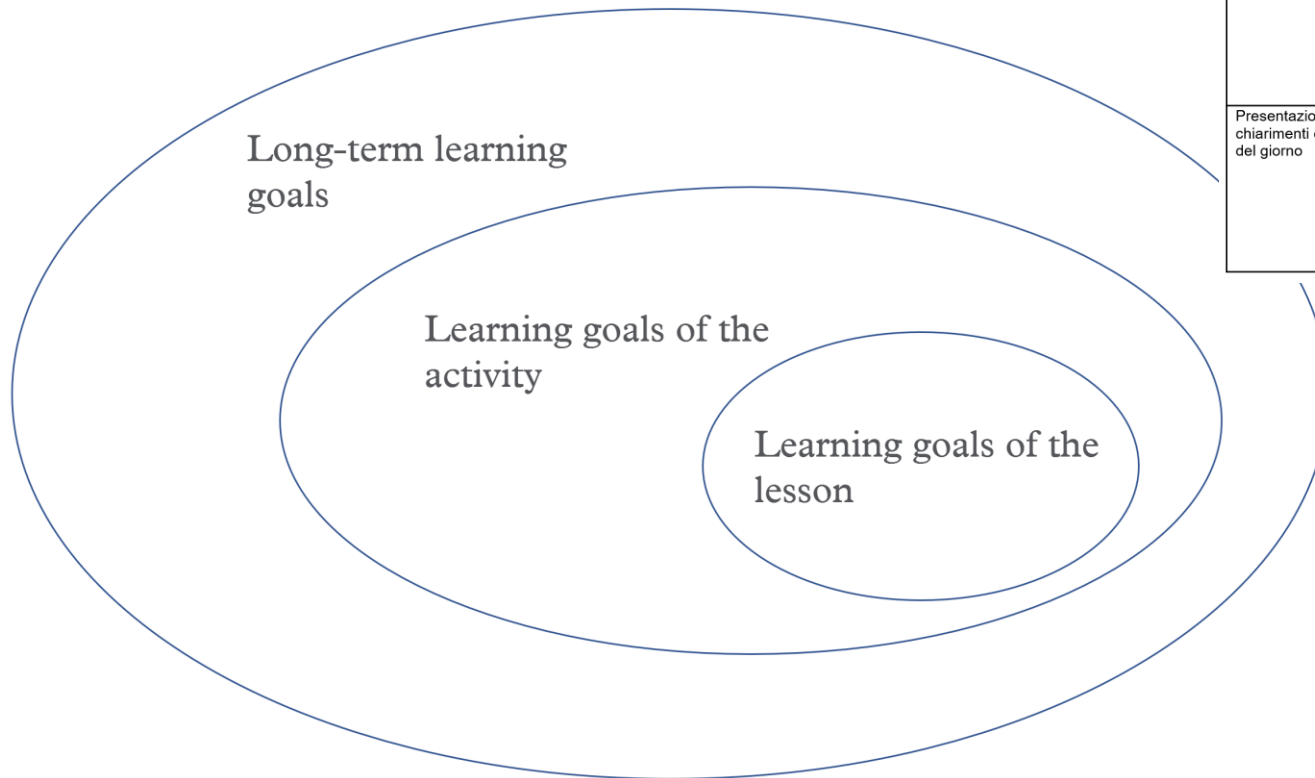
Some problems are beyond our (or the teachers') capabilities.



This suggests the need to involve other institutional actors, such as headmasters or the Ministry itself.

# Some solutions

Some “misunderstandings” are easier to tackle.



Presentazione della lezione (matematica)					
Descrizione dell'attività	Consegna e/o domande del docente	Reazioni degli studenti e indicazioni per il docente	Raggruppamenti	Tempi	Intenzionalità educative (i perché delle scelte)
Introduzione alla lezione e presentazione dell'argomento	<i>(riepilogo alla classe, da parte dell'insegnante, delle attività già svolte e specifica del topic del giorno)</i>		<input type="checkbox"/> Grande gruppo <input type="checkbox"/> Piccolo gruppo <input type="checkbox"/> Coppia <input type="checkbox"/> Individuale <i>(eventualmente anche elencazione dei gruppi e delle motivazioni)</i>		
Controllo dei compiti <i>(opzionale)</i>					
Formulazione/consegna del problema del giorno					<i>(esplicitazione delle problematiche che si vogliono mettere in luce)</i>
Presentazione / chiarimenti del problema del giorno					

# Overall achievements

In particular, as researchers and educators:

- We are more aware of our role in communicating with the teachers about foreign practices;
- We have a «toolbox» of possible actions for the implementations of other foreign practices;
- We can contribute to similar project in other contexts.





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How do you see  
Lesson Study  
in your context?



ありがとうございました

(Thank you very much!)